JULIAN LADERMAN

USEFUL PROBABILITY FOR BRIDGE PLAYERS

Master Point Press
214 Merton St. Suite 205
Toronto, Ontario, Canada
M4S 1A6 (647)956-4933

Email: info@masterpointpress.com
Websites: www.masterpointpress.com
www.teachbridge.com
www.bridgeblogging.com
www.ebooksbridge.com

Library and Archives Canada Cataloguing in Publication

Title: Useful probability for bridge players / Julian Laderman.
Names: Laderman, Julian, author.
Identifiers: Canadiana (print) 2020027225X | Canadiana (ebook) 20200272292 | ISBN 9781771400596 (softcover) | ISBN 9781771405539 (PDF) | ISBN 9781554947003 (HTML) | ISBN 9781771408998 (Kindle)
Subjects: LCSH: Contract bridge. | LCSH: Probabilities.
Classification: LCC GV1282.3 .L326 2020 | DDC 795.41/5—dc23

Editor Ray Lee
Copy editor/Interior format Sally Sparrow
Cover design Olena S. Sullivan/New Mediatrix

1 2 3 4 5 6 7 23 22 21 20

Contents

Pages with Important Reference Tables

Chapter 1

Who Needs Bridge Probability?

Experience is a Great Teacher

Often at a bridge table an opponent who is aware of my prior professional life comments, 'I hated mathematics. I'm sure glad that playing bridge does not require any knowledge of mathematics.' I immediately look for the nearest Director and estimate the approximate time he needs to cover the distance to my table. This will dictate how lengthy a response I can provide before hearing 'Julian, stop talking! The round has been called.' Sadly, probably only for me, there is never enough time. This book provides me with the opportunity to give a full retort.

Probability is the mathematics of studying how likely an event is to occur. All experienced bridge players have a significant amount of knowledge in this field, whether they realize it or not. Let me demonstrate by asking you four quick questions:

1) Are you more likely to be dealt 14 high-card points (HCP) or 21 HCP?
2) Your partner opens 1♥. Is he more likely to hold five hearts or six hearts?
3) You open the bidding 1♠. Is your partner more likely to have three spades or a void in spades?
4) You open the bidding 1NT. Is your partner more likely to have 6 HCP or 1 HCP?

The first option was the correct answer to each of the four questions. Let's hope you just got a 100% on this probability test. You were using the experience you gained from playing thousands of bridge hands to estimate likelihood. Your performance was likely far better than that of a person equipped with a Ph.D. in mathematics who has never played bridge. Be proud!

One way to measure the likelihood of an event occurring is to measure its relative frequency. This is obtained by simply recording how often an event occurs in a large number of trials.

$$\text{Relative frequency} = \frac{\text{number of times event occurs}}{\text{number of trials}}$$

Suppose a baseball player has had 423 official times at bat and has 132 hits. His batting average is obtained by dividing 132 by 423. The resulting value of 0.312 (or 31.2%) is very impressive and his agent will certainly use it when negotiating the player's next contract. The baseball world ignores the decimal point; the batting average is 312.

Even though you have not kept track of bridge hands* that you have played over the years, you were able to use your experience to answer the above questions. Of course, it would have been much more difficult to estimate the actual likelihood of each of the events rather than to compare the two.

For example, suppose you wish to know the likelihood of being dealt a hand where the longest suit has five or more cards. If you kept track of your next 1000 hands, and recorded how many had at least one suit with five or more cards, you would obtain an extremely accurate estimate of the true value. Please don't carry out this experiment — it would be foolish, since it is very easy to calculate the likelihood of the event. I won't leave you hanging: it is 65%.

In a very early bridge lesson, a beginner is told that the number 26 is special. With a combined 26 high-card points (HCP) a

---

* Bridge vocabulary is a little vague at times. Consider the words 'hand' and 'deal'. A bridge deal consists of all four bridge hands. The word hand can refer either to the thirteen cards a player is dealt or to all four hands. Sometimes the word 'hand' and 'deal' overlap in usage. At the end of a session, we often remind our partners to pick up 'hand records' so that they can be discussed over dinner. It would be more accurate to refer to those sheets as 'deal records'.

partnership has enough good cards that they can probably make a game in notrump or a major. Of course, any bridge player can construct freakish pairs of hands with a combined 30 HCP that cannot make game and pairs with a combined 20 HCP that can. The 26 HCP number is based on the experience of top players; this kind of guideline or maxim allows new players to learn bridge probability more quickly than by relying on their own personal experience. Our bidding systems are based on this and similar maxims. In Chapter 15, we will look at the maxims that relate to bidding and see how bidding systems are designed to communicate the information necessary for a player to employ these maxims.

Bridge players are armed with many probability-based maxims for both bidding and play. I cringe slightly when they are referred to as rules (or even laws) rather than guidelines. We will be looking at many of them throughout the book in order to learn when they are useful and when they may be misleading.

A one-sentence answer to the question posed in the title of this chapter: *You greatly need it, and you already know much more than you think.*

One Can Be Fooled by Experience

After extolling the virtues of intuition generated through experience, I must point out that it often can be quite misleading. Let me add one more question to the earlier four.

Is the next bridge hand you are dealt more likely to be:

♠ A 7 5 4 ♥ K 8 5 ♦ A 8 7 2 ♣ A 6

or ♥ A K Q J 10 9 8 7 6 5 4 3 2 and void in the other suits?

In fact, both are equally likely to be your next hand. It is tempting but wrong to believe that the first hand is more likely to occur. The first hand feels like a common hand whereas the second hand is extremely remarkable. If you were actually dealt the second hand, you would be telling all your friends, even your non-bridge playing friends, 'You won't believe the hand that I was dealt!'

Your intuition can be led astray since all players have been dealt hundreds of hands that are very similar to the first hand. An alternative argument is: since hands with 10 HCP are certainly more likely than 15 HCP hands, a few players (very few) may mistakenly believe that the 10 HCP hand with the thirteen-card heart suit is more likely than its alternative with 15 HCP. But now let's suppose the specific spot cards were replaced by x, so that the comparison is between:

♠ A x x x ♥ K x x ♦ A x x x ♣ A x

and ♥ A K Q J x x x x x x x x x

Now, the first hand would be more than two million times more likely to be dealt than the second. Really! I am not kidding.†

Let's return to the original example with specific spot cards. *Any two hands where all thirteen cards are specified are equally likely to occur.* You will almost certainly live your life without being dealt either of the above specific hands. There are just too many possible bridge hands. There are, would you believe, 635 billion different thirteen-card hands. The United States, Canada, Mexico, and Bermuda have a combined population of 498 million‡. This means that if every person (including one-day-old babies) in these countries dealt themselves 1000 hands, and even if miraculously no hand were repeated, it would still be impossible for them to deal out all possible thirteen-card bridge hands. So, if you are ever dealt thirteen hearts, you should expect that a 'friend' set it up as a joke and that an opponent is holding thirteen spades.

When intuition is developed through experience, it is much better at common situations than at rare ones. Extremely excellent hands are not very common. Suppose you were asked whether you are more likely to be dealt a hand with exactly 23 HCP or a hand with more than 23 HCP. You should expect to be dealt a hand with exactly 23 HCP only about once every 900 hands. If you combine

† This statement will be justified in Chapter 20. Here, I was just showing off.

‡ You may be surprised that I included the 65,000 residents of Bermuda – I wanted to include all regions that hold ACBL sanctioned events.

all hands in the range from 24 HCP up to 37 HCP (maximum possible), taken together their chance of occurring is still less than the case of exactly 23 HCP. Anything above 23 is in very rarefied air.

A primary goal of this book is to build on your good intuition about bridge probability and point out where you could be fooled by your bad intuition. My hope is that I can modify your bad intuition. The book uses the philosophy that *usefulness* trumps *theory*. This is described in the next section. Similarly, there is a philosophy that *approximation* trumps *exactness*. At the bridge table, approximation is the best that one can hope to achieve. This will be described at the end of this chapter.

Useful Topics and Not So Useful Topics

The book tries to be as gentle as possible with the mathematics. After all, if you cannot understand a topic, or if a result is not compatible with your intuition, the information will not be useful. There are different levels of understanding. It is one thing to understand what you read in a book, and another thing to be able to comfortably apply the knowledge a year from now. The latter requires a much deeper understanding. A procedure that goes against a person's intuition has a very short mental shelf life.

The main goal most players will have in reading this book is to improve their decision-making at the table. Decisions fall into two broad categories. Some are made while actually playing bridge, and others are made between sessions. For example, when forming a new partnership, dozens of decisions about bidding conventions and signaling must be resolved while filling out a convention card. Deciding what system and which conventions to play may be influenced by how frequently certain types of hands occur.

Let's first consider decisions that are made during the bidding or play. These are situations where a knowledge of probability can help you to make the right choice between alternative bids, or between alternative lines of play, or between alternative ways to play a suit combination. The calculations that can be done are obviously very limited — one cannot use a calculating device, and even difficult bridge decisions must be made in a minute or two at most. Not an environment conducive for doing mathematics.

Between sessions you can take all the time you wish; you can even employ the great computational power hidden in your Smartphone. Even though it is clearly too late to change results from your last session, you may be able to learn something that can be applied in similar situations in the future. During a session, it is usually wise for partnerships not to modify the bidding systems, conventions, and signaling methods they are playing. Between sessions, they can discuss changes at their leisure.

When you are driving a car over a bridge, you need only know how to drive and be able to see the road. It is unnecessary to know the engineering and mathematics that enable the bridge to stand. You very reasonably take it for granted. The mathematics of playing bridge is very different. The decision-making issues considered in this book have a probabilistic basis. I agree that fully understanding the details as to how certain mathematical values can be computed, or at least approximated, is not directly useful. However, this knowledge puts you in a better position to trust the results (useful stuff) and thereby use them correctly. It also gives you the courage not to abandon a correct principle just because it leads to a poor outcome on a particular hand. So even though these sections of the book are not directly useful, they may indirectly be very useful.

Questions related to how likely a particular type of hand is to occur are only moderately useful. We will examine such questions even though they are sometimes merely 'intellectual curiosities' rather than helpful decision tools. For example, the problem in the previous section about the probability that the longest suit dealt has five or more cards is not directly useful. For any hand, as soon as you look at your cards, you will know with complete certainty whether or not you actually have a five-card suit. But as already stated, knowing the likelihood of certain types of hands can be useful when designing bidding systems as well as when deciding which conventions to adopt.

Throughout this book, chapters will be given a *usefulness number*, between 1 and 10. Each chapter will start with a description of what is useful in the chapter and what is not so useful, and that description will indicate the reason behind the number. You may choose just to read the useful results and skip the sections where the underlying mathematics is developed. Typically, general mathematics books first present the underlying mathematics and then show how it can be used to derive important results. I will often first present the results, and then you can decide whether you care to see how they were obtained. Sometimes the complete explanation might not come for a dozen chapters, but in practically all cases, it will appear, I promise! By the end of the book you will feel comfortable that I did not just make up the number 635 billion for the number of possible hands.

Sadly, I have to accept the reality that the first half of any of my books is read more than the second half. I realize that I should not take it personally since it is true of all books, and particularly books involving mathematics. This is part of the rationale for presenting results before getting weighed down in the supporting mathematics. That is the key to the distinction between Part II and Part III. Especially in Chapters 19 to 21, you will learn how certain results and values are obtained (I hope you will still care).

Even the chapter on the history of probability is not totally useless. Games are not 'just' games, and gambling was the early motivation for the development of probability theory. Mathematicians and gamblers have had a wonderful parasitic relationship for four centuries.

Approximating, Assuming, and even Pretending

In a mathematics course, accuracy is paramount. I was not impressed by my students when they would tell me that they used estimation to obtain a reasonable answer. However, for bridge players, exactness is overrated.

During the play of a hand, approximation is an essential tool for all probability-related decisions. It is the only way around the constraints of time and not having computing devices. After considering the information learned from the prior bidding and play, you base your decisions either on approximate values that you have memorized, or intuition gained from experience, or a few probability-related bridge maxims.

A well-known bridge maxim is 'With an odd number of cards missing, expect them to split as evenly as possible. With an even number of cards missing, expect them to split unevenly'. Another favorite is 'Eight ever, nine never'. These maxims are themselves very crude approximations that summarize and simplify the actual probabilities. When we look at these maxims, I will air my objections and provide alternatives that result in superior approximate values.

This chapter started off with a probability quiz. You were asked which of two possibilities was more likely. The answers did not require any computation. This type of comparison often occurs when making bridge decisions. A player wishes to know which option is more likely to be successful. It is not important whether Option 1 will be successful 90% of the time and Option 2 only 70% of the time or whether Option 1 will be successful 40% of the time and Option 2 only 25% of the time. With either scenario, Option 1 should be selected.

In this book, I have tried to minimize the inclusion of the tables of numerical entries. However, I did have to include tables for suit splits, shape of hands, high-card points, and joint partnership holdings. The values in these tables are essential. At the start of this book, after the Contents page, I provide page references to six important tables. When a table is introduced, I try to provide an explanation of values that would go against a bridge player's intuition and I try to modify that intuition. I did not compute all the values myself since they have appeared in many other whist and bridge books. However, when using values from tables it is important to always employ some approximation technique, such as your intuition from experience. Ask yourself if the value you find in a table is consistent with your expectation for that value. If so, great; if not, ask yourself whether your intuition is wrong or is the value wrong?

Thankfully, an incorrect value in a table is often significantly different from the correct value. The greater the error, the easier that error is to spot. Suppose table values involve percentages that should logically add up to 100%. Due to rounding of values it is reasonable for the values to add up to some value in the range of 97% to 103%. At times, I have cheated when rounding in this book in order to force the values to add up to 100%. But should you ever encounter a table of this sort that adds up to something like 87% or 114%, you know something is wrong!

There are many ways the values in a table can fail the 'smell test'. Suppose you look at a table for the total HCP of a partnership. It contains an entry for the probability that the North-South partnership is dealt 23 HCP and East-West is dealt 17 HCP. However, it indicates a different value for the probability that North-South are dealt 17 HCP and East-West are dealt 23. Even though you don't know the actual value of either, you know both events should have the same probability of occurring. If they differ, one or both of the entries must be an error. Sometimes you can be certain of a pattern in a table. One would expect the entry for the probability that North-South are dealt 28 HCP to be less than that for 27 HCP. Likewise, holding a combined 27 HCP is less likely than 26. If the values are not decreasing when North-South hold more than 20 HCP, something is wrong.

Since one of my major goals is to make you more comfortable using the results, at times I will resort to less accurate techniques that enable me to justify approximate results. Often I am rounding off values. Sometimes I am slightly shading the truth in order to obtain values and methods that are easier for bridge players to either remember or understand.

There are three general assumptions that are necessary to analyze bridge probability:

1) Assume that the cards, whether they are dealt by hand or generated by a computer, achieve randomness. In Chapter 6, we will see that dealt cards often don't totally achieve that goal. In Chapter 22, we will see a method to test computer-generated hands.
2) Assume players always adhere to standard bridge rules. If a player does not follow suit, you are certain he has no cards in the suit. We are assuming no carelessness or cheating.

3) Assume players are always trying to win. They will play to the best of their ability. This is actually implied by Assumption 2.

Many other specific bridge bidding assumptions are necessary. Consider problems such as: My partner is the dealer, what is the probability that he will open the bidding 2♠? The answer involves assumptions about what type of hand would be appropriate for a 2♠ bid. What is your point range for that bid? Could it be done on a five-card spade suit? Could it be done when holding a four-card heart suit? One must make many assumptions.

In summary, questions such as computing the probability of being dealt a hand with exactly seven hearts can be answered with exactness. No assumptions are necessary since it does not involve anything a player does. But questions related to the bidding or play of a hand require assumptions about player decisions.

At times, the power of assuming and approximating will border on pretending. My goal is to display the mathematics. At times, the arithmetic is easier with questionable assumptions. You don't have to live with my bridge assumptions since you can form your own assumptions and apply similar mathematics.

Sometimes, after explaining when a mathematical principle should not be used, I will go ahead and use it anyway. I am not being stubborn. I will make it clear when I am doing this and why it is useful for me to pretend that what I am doing is acceptable. In Appendix 6, I will summarize my pretending.

Throughout this book I will be assuming that readers have never taken a course that even touched on the topic of probability. My goal is to write a book that is appropriate for bridge players regardless of their background in mathematics, even a Ph.D. in mathematics. I am further doubling down by trying to write a book for all levels of bridge ability.

I certainly feel safe assuming that all readers are bridge players. Who else would read this book?

GENERAL INTEREST

Beginners rely heavily on bridge maxims. Are they accurate? What is the mathematics behind them? *Useful Probability for Bridge Players* examines these questions. The emphasis here is on 'useful'. This is not an academic tome, but a discussion of the aspects of probability that every bridge player needs to know and understand. Topics include suit splits, suit combinations, percentage plays, the Principle of Restricted Choice, choosing bidding systems/conventions and the application of probability to bidding decisions.

– Praise for *A Bridge to Inspired Declarer Play* –

"... [Laderman] gives the most comprehensible explanation of probabilities I can remember reading."
The New York Times (Dec, 2009) Phillip Alder

JULIAN LADERMAN, Ph.D (New York) is a retired applied mathematics professor (CUNY) who specializes in making difficult concepts clear. He is the author of *A Bridge to Simple Squeezes* and *A Bridge to Inspired Declarer Play*, both ABTA Book of the Year award winners. His bridge history book, *Bumblepuppy Days*, won the IBPA Alan Truscott Memorial Award.

MASTER POINT PRESS
978-177140-059-6